MA010 Graph Theory—Homework Set #1

Each problem is worth 4 points. The solutions should be submitted using the university information system by **October 24**. Please submit your solutions as pdf files produced by a suitable text editor, e.g., LaTeX; solutions that are not submitted as pdf files may be assigned zero points. Your solutions should contain references to all sources, including those available on the web, that you have used.

1. Prove that a graph G with at least three vertices is a tree if and only if G is not a complete graph and adding any edge to G creates exactly one cycle.

2. Prove that an n-vertex graph G with k connectivity components has at least $n - k$ edges.

3. Prove the following statement: if G is an n-vertex plane graph with m edges, f faces and k connectivity components, then $n + f = m + k + 1$.

4. Prove that every quadrangulation of the plane is 2-colorable (recall that a quadrangulation is a plane graph such that each face is bounded by a cycle of length four).