MA010 Graph Theory—Homework Set #2

Each problem is worth 4 points. The solutions should be submitted using the university information system by **December 5**. Please submit your solutions as pdf files produced by a suitable text editor, e.g., LaTeX; solutions that are not submitted as pdf files may be assigned zero points. Your solutions should contain references to all sources, including those available on the web, that you have used.

1. Show that every connected chordal graph with $n \geq 2$ vertices has at most $n-1$ cliques (recall that a clique is an *inclusion-wise maximal* complete subgraph).

2. Prove the following statement. Let G be a graph and let G_1 and G_2 be spanning subgraphs of G such that each edge of G belongs to either G_1 or G_2. Show that there are k_1 components of G_1 and k_2 components of G_2 for some $k_1 + k_2 \leq \alpha(G)$ such that each vertex of G belongs to (at least) one of the $k_1 + k_2$ components.
   Hint: Use König's Theorem.

3. Show that the chromatic index of any tree with maximum degree Δ is Δ.

4. Show that a graph G is 2-edge-connected if and only if G is connected and every edge is contained in a cycle.