IA168 Algorithmic Game Theory

Tomáš Brázdil

Organization of This Course

Sources:

- Lectures (slides, notes)
  - based on several sources
  - slides are prepared for lectures, some stuff on greenboard ($\Rightarrow$ attend the lectures)

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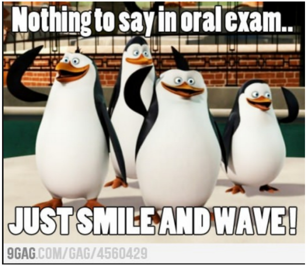
- Books:
  - Nisan/Roughgarden/Tardos/Vazirani, **Algorithmic Game Theory**, Cambridge University, 2007.
    Available online for free:
    http://www.cambridge.org/journals/nisan/downloads/Nisan_Non-printable.pdf
  - Tadelis, **Game Theory: An Introduction**, Princeton University Press, 2013

(I use various resources, so please, attend the lectures)

Evaluation

- **Oral exam**
- Homework



  - 3 homework assignments

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An example of an instruction email (from another course with the same system):

```
It is typically not sufficient to devote a single
afternoon to the preparation for the exam.
You have to know _everything_ (which means every
single thing) starting with the slide 42
and ending with the slide 245 with notable exceptions
of slides: 121 - 123, 137 - 140, 165, 167.
Proofs presented on the whiteboard are also mandatory.
```

Most importantly,

The previous slide is not a joke!

What is Algorithmic Game Theory?

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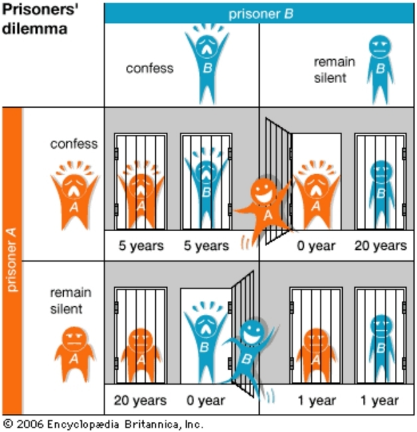
What does the "algorithmic" mean?

- It means that we are "concerned with the computational questions that arise in game theory, and that enlighten game theory. In particular, questions about finding efficient algorithms to 'solve' games."

Let's have a look at some examples ....

Prisoner's Dilemma



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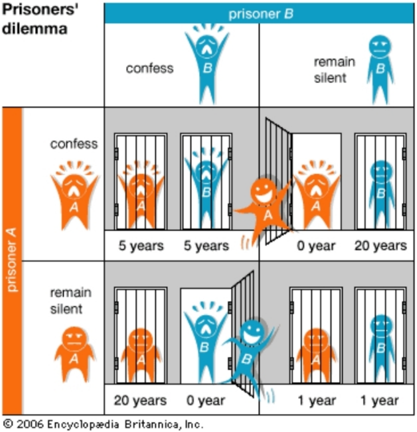

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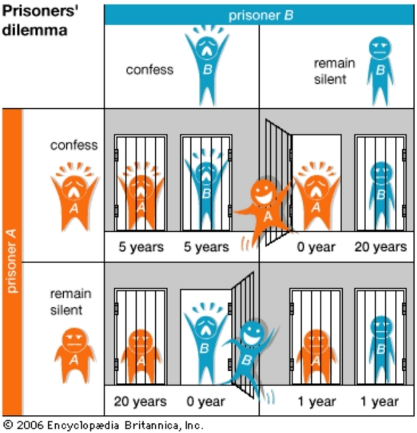

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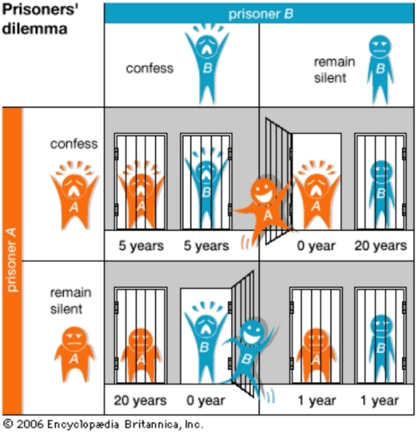

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The problem: What would the suspects do?

Prisoner's Dilemma – Solution(?)

	C	S
C	$-5, -5$	$0, -20$
S	$-20, 0$	$-1, -1$

Rational "row" suspect (or his adviser) may reason as follows:

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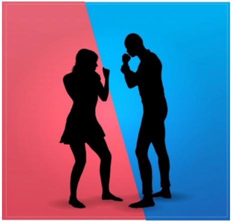
Are there always "dominant" strategies?

Nash equilibria – Battle of Sexes



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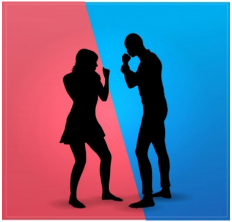

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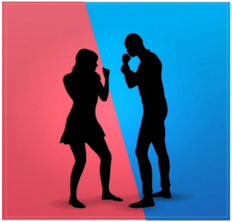

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If they cannot communicate, where should they go?

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Battle of Sexes can be modeled as a game of two players (the couple) with the following payoffs:

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(O, O) is an example of a *Nash equilibrium* (as is (F, F))

Mixed Equilibria – Rock-Paper-Scissors

	R	P	S
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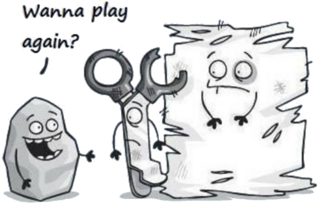

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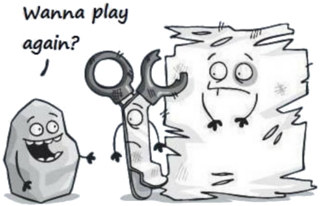

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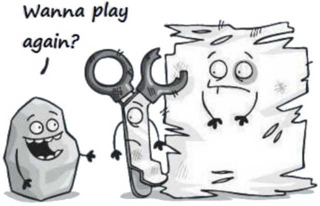

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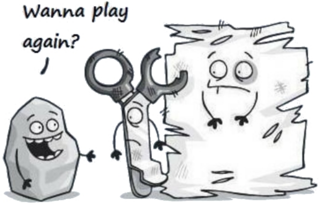

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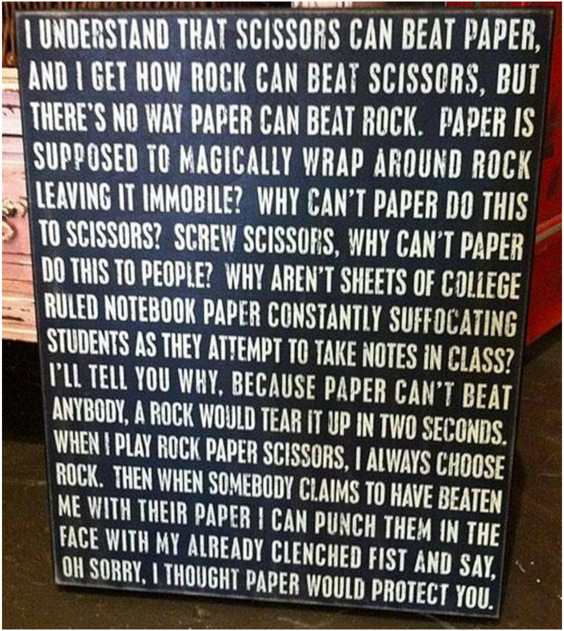
  Use *mixed strategies*: Each player plays each pure strategy with probability 1/3. The expected payoff of each player is 0 (even if one of the players changes his strategy, he still gets 0!).

Philosophical Issues in Games



Dynamic Games

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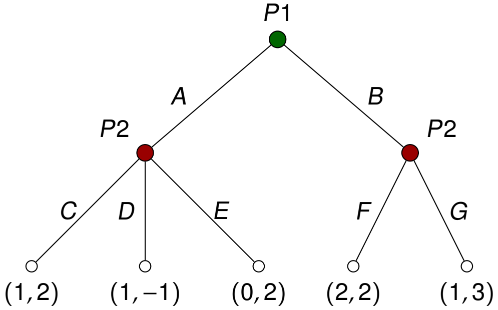

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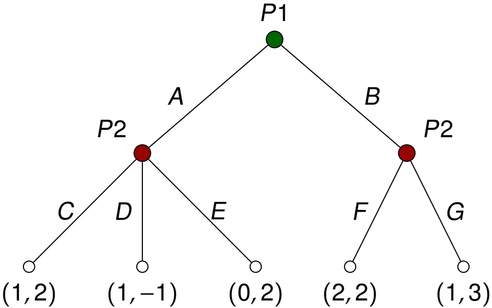

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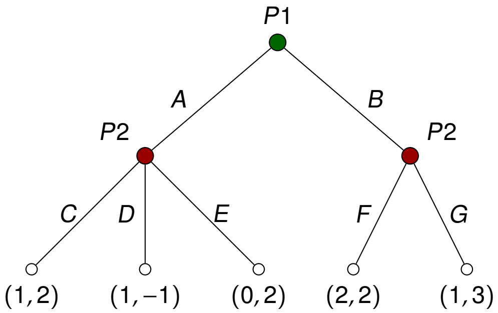

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What is their relationship to the strategic form games?

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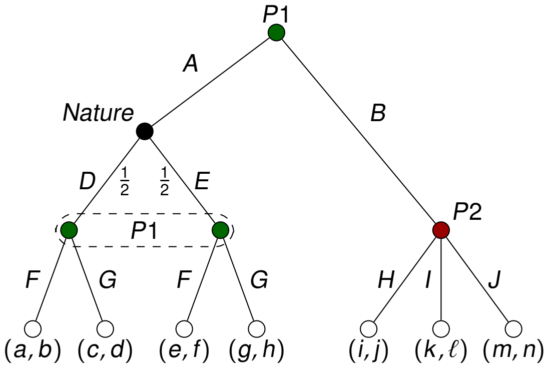

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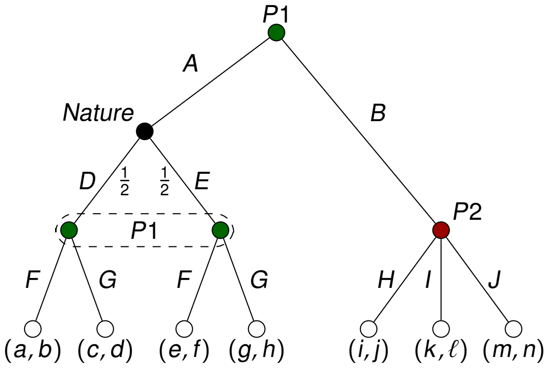

Again, how to solve such games?

Games of Incomplete Information

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In all previous games the players knew all details of the game they played, and this fact was a “common knowledge”. This is not always the case.

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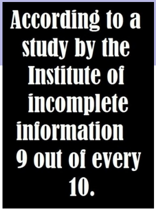

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$$u_1(b_1, b_2) = \begin{cases} v_1 - b_1 & b_1 > b_2 \\ \frac{1}{2}(v_1 - b_1) & b_1 = b_2 \\ 0 & b_1 < b_2 \end{cases}$$

Here v_1 is the private value that player 1 assigns to the item and so the player 2 **does not know** u_1**.**

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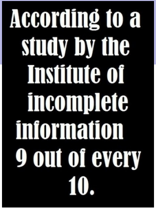

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How to deal with such a game? Assume the "worst" private value? What if we have a partial knowledge about the private values?

Inefficiency of Equilibria

In Prisoner's Dilemma, the selfish behavior of suspects (the Nash equilibrium) results in somewhat worse than ideal situation.

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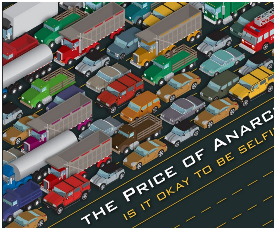
Price of Anarchy is the maximum ratio between values of equilibria and the value of an optimal solution.

Inefficiency of Equilibria – Selfish Routing

Consider a transportation system where many agents are trying to get from some initial location to a destination. Consider the welfare to be the average time for an agent to reach the destination. There are two versions:



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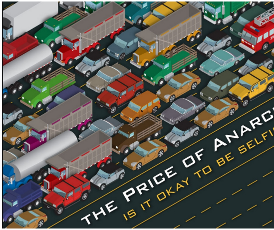

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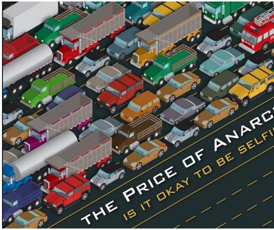

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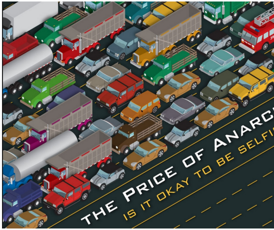

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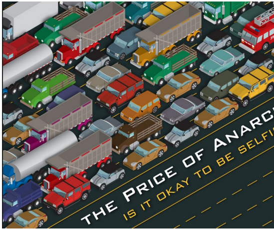
Price of Anarchy measure the ratio between average travel time in these two cases.

Inefficiency of Equilibria – Selfish Routing

Consider a transportation system where many agents are trying to get from some initial location to a destination. Consider the welfare to be the average time for an agent to reach the destination. There are two versions:



- “Centralized”: A central authority tells each agent where to go.
- “Decentralized”: Each agent selfishly minimizes his travel time.

Price of Anarchy measure the ratio between average travel time in these two cases.

Problem: Bound the price of anarchy over all routing games?

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- Games in Logic: modal and temporal logics, Ehrenfeucht-Fraisse games, etc.

Games in Computer Science

Games, the Internet and E-commerce: An extremely active research area at the intersection of CS and Economics

Basic idea: “The internet is a HUGE experiment in interaction between agents (both human and automated)”

How do we set up the rules of this game to harness “socially optimal” results?

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- Remaining time will be devoted to selected topics from extensive form games, games on graphs etc.

Static Games of Complete Information

Strategic-Form Games

Solution concepts

Static Games of Complete Information – Intuition

Proceed in two steps:

1. Players *simultaneously and independently* choose their *strategies*. This means that players play without observing strategies chosen by other players.

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A fact E is a *common knowledge* among players $\{1, \ldots, n\}$ if for every sequence $i_1, \ldots, i_k \in \{1, \ldots, n\}$ we have that i_1 knows that i_2 knows that ... i_{k-1} knows that i_k knows E.

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The goal of each player is to maximize his payoff (and this fact is a common knowledge).

Strategic-Form Games

To formally represent static games of complete information we define *strategic-form games*.

Definition 2

A game in *strategic-form* (or normal-form) is an ordered triple $G = (N, (S_i)_{i \in N}, (u_i)_{i \in N})$, in which:

- $N = \{1, 2, \ldots, n\}$ is a finite set of *players*.
- S_i is a set of *(pure) strategies* of player i, for every $i \in N$.

  A *strategy profile* is a vector of strategies of all players $(s_1, \ldots, s_n) \in S_1 \times \cdots \times S_n$.

  We denote the set of all strategy profiles by $S = S_1 \times \cdots \times S_n$.
- $u_i : S \to \mathbb{R}$ is a function associating each strategy profile $s = (s_1, \ldots, s_n) \in S$ with the *payoff* $u_i(s)$ to player i, for every player $i \in N$.

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Definition 3

A *zero-sum* game G is one in which for all $s = (s_1, \ldots, s_n) \in S$ we have $u_1(s) + u_2(s) + \cdots + u_n(s) = 0$.

Example: Prisoner's Dilemma

- $N = \{1, 2\}$
- $S_1 = S_2 = \{S, C\}$
- u_1, u_2 are defined as follows:
  - $u_1(C, C) = -5$, $u_1(C, S) = 0$, $u_1(S, C) = -20$, $u_1(S, S) = -1$
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(Is it zero sum?)

We usually write payoffs in the following form:

	C	S
C	$-5, -5$	$0, -20$
S	$-20, 0$	$-1, -1$

or as two matrices:

	C	S
C	-5	0
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	C	S
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Strategic-form game model $(N, (S_i)_{i \in N}, (u_i)_{i \in N})$

- $N = \{1, 2\}$
- $S_i = [0, \infty)$
- $u_1(q_1, q_2) = q_1(\kappa - q_1 - q_2) - q_1 c_1$
  $u_2(q_1, q_2) = q_2(\kappa - q_1 - q_2) - q_2 c_2$

Solution Concepts

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Example 4

Nash equilibrium is a solution concept. That is, we "solve" games by finding Nash equilibria and declare them to be reasonable outcomes.

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Here 4. implies non-cooperative game theory: Each player is in control of his actions, and he will stick to an action only if he finds it to be in his best interest.

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- iterated elimination of strictly dominated strategies (IESDS)
- rationalizability
- Nash equilibria

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For now, let us concentrate on

pure strategies only!

I.e., no mixed strategies are allowed. We will generalize to mixed setting later.

Notation

- Let $N = \{1, \ldots, n\}$ be a finite set and for each $i \in N$ let X_i be a set. Let $X := \prod_{i \in N} X_i = \{(x_1, \ldots, x_n) \mid x_j \in X_j, j \in N\}$.
  - For $i \in N$ we define $X_{-i} := \prod_{j \neq i} X_j$, i.e.,

    $$X_{-i} = \{(x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n) \mid x_j \in X_j, \forall j \neq i\}$$

  - An element of X_{-i} will be denoted by

    $$x_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$$

    We slightly abuse notation and write (x_i, x_{-i}) to denote $(x_1, \ldots, x_i, \ldots, x_n) \in X$.

Strict Dominance in Pure Strategies

Definition 5

Let $s_i, s_i' \in S_i$ be strategies of player i. Then s_i' is *strictly dominated* by s_i (write $s_i \succ s_i'$) if for any possible profile of the other players' strategies, $s_{-i} \in S_{-i}$, we have

$$u_i(s_i, s_{-i}) > u_i(s_i', s_{-i}) \quad \text{for all } s_{-i} \in S_{-i}$$

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Claim 1

An intelligent and rational player will never play a strictly dominated strategy.

Clearly, intelligence implies that the player should recognize dominated strategies, rationality implies that the player will avoid playing them.

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Corollary 8

If the strictly dominant strategy equilibrium exists, it is unique and rational players will play it.

Examples

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In the Battle of Sexes:

	O	F
O	$2, 1$	$0, 0$
F	$0, 0$	$1, 2$

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In the Battle of Sexes:

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no strictly dominant strategies exist.

Indiana Jones and the Last Crusade

(Taken from Dixit & Nalebuff's "The Art of Strategy" and a lecture of Robert Marks)

Indiana Jones, his father, and the Nazis have all converged at the site of the Holy Grail. The two Joneses refuse to help the Nazis reach the last step. So the Nazis shoot Indiana's dad. Only the healing power of the Holy Grail can save the senior Dr. Jones from his mortal wound. Suitably motivated, Indiana leads the way to the Holy Grail. But there is one final challenge. He must choose between literally scores of chalices, only one of which is the cup of Christ. While the right cup brings eternal life, the wrong choice is fatal. The Nazi leader impatiently chooses a beautiful gold chalice, drinks the holy water, and dies from the sudden death that follows from the wrong choice. Indiana picks a wooden chalice, the cup of a carpenter. Exclaiming "There's only one way to find out" he dips the chalice into the font and drinks what he hopes is the cup of life. Upon discovering that he has chosen wisely, Indiana brings the cup to his father and the water heals the mortal wound.

Indiana Jones and the Last Crusade (cont.)

Indy Goofed

- Although this scene adds excitement, it is somewhat embarrassing that such a distinguished professor as Dr. Indiana Jones would overlook his dominant strategy.
- He should have given the water to his father without testing it first.
  - If Indiana has chosen the right cup, his father is still saved.
  - If Indiana has chosen the wrong cup, then his father dies but Indiana is spared.
- Testing the cup before giving it to his father doesn't help, since if Indiana has made the wrong choice, there is no second chance – Indiana dies from the water and his father dies from the wound.

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Thus, everyone knows that nobody will play strictly dominated strategies in the smaller game (and such strategies may indeed exist).

Because it is common knowledge that all players will perform this kind of reasoning again, the process can continue until no more strictly dominated strategies can be eliminated.

IESDS

The previous reasoning yields the **Iterated Elimination of Strictly Dominated Strategies (IESDS)**:

Define a sequence $D_i^0, D_i^1, D_i^2, \ldots$ of strategy sets of player i.
(Denote by G_{DS}^k the game obtained from G by restricting to D_i^k, $i \in N$.)

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Remark: If all S_i are *finite*, then in 2. we may remove only some of the strictly dominated strategies (not necessarily all). The result is *not* affected by the order of elimination since strictly dominated strategies remain strictly dominated even after removing some other strictly dominated strategies.

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all strategies survive all rounds (i.e. IESDS $\equiv$ anything may happen, sorry)

A Bit More Interesting Example

	L	*C*	*R*
L	4, 3	5, 1	6, 2
C	2, 1	8, 4	3, 6
R	3, 0	9, 6	2, 8

IESDS on greenboard!

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Hotelling (1929) and Downs (1957)

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  (Here 10 means ten percent in the real-world)
- Voters vote for the closest candidate. If there is a tie, then $\frac{1}{2}$ got to each candidate
- Payoff: The number of voters for the candidate; each candidate (selfishly) strives to maximize this number

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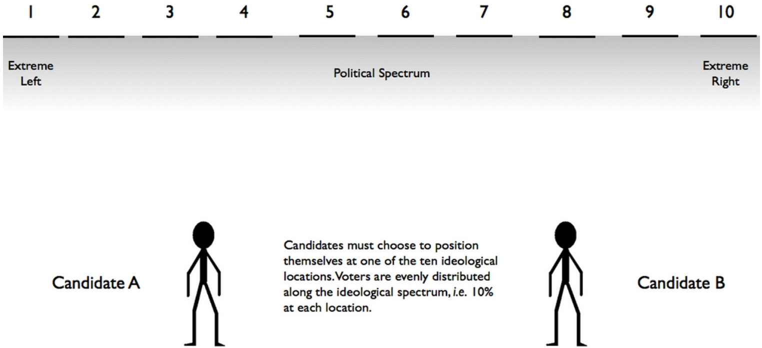

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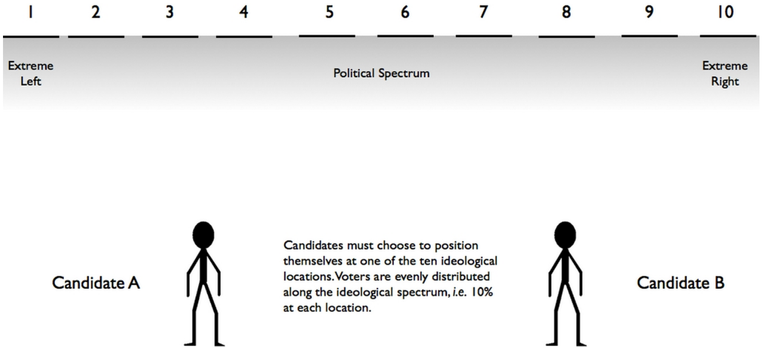

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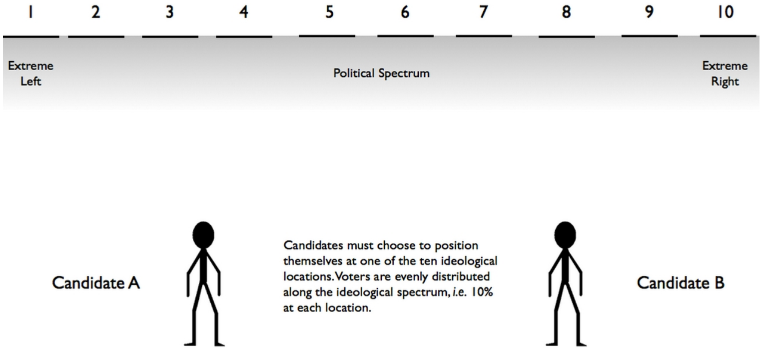

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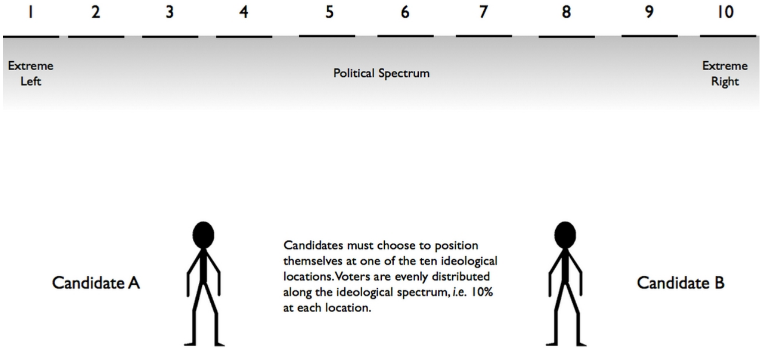

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- $\ldots$
- only $5, 6$ survive IESDS

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IESDS eliminated apparently unreasonable behavior (leaving "reasonable" behavior implicitly untouched).

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Let us formalize this type of reasoning...

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A strategy $s_i \in S_i$ of player i is a *best response* to a belief $s_{-i} \in S_{-i}$ if

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A rational player never plays any strategy that is never best response.

Best Response vs Strict Dominance

Proposition 1

If s_i is strictly dominated for player i, then it is never best response.

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The opposite does not have to be true in pure strategies:

	X	Y
A	$1, 1$	$1, 1$
B	$2, 1$	$0, 1$
C	$0, 1$	$2, 1$

Here A is never best response but is strictly dominated neither by B, nor by C.

Elimination of Stupid Strategies = Rationalizability

Using similar iterated reasoning as for IESDS, strategies that are never best response can be iteratively eliminated.

Define a sequence $R_i^0, R_i^1, R_i^2, \ldots$ of strategy sets of player i.
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(Warning: For some reasons, rationalizable strategies are almost always defined using mixed strategies!)

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all strategies are rationalizable.

Cournot Duopoly

$G = (N, (S_i)_{i \in N}, (u_i)_{i \in N})$

- $N = \{1, 2\}$
- $S_i = [0, \infty)$
- $u_1(q_1, q_2) = q_1(\kappa - q_1 - q_2) - q_1 c_1 = (\kappa - c_1)q_1 - q_1^2 - q_1 q_2$
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Assume for simplicity that $c_1 = c_2 = c$ and denote $\theta = \kappa - c$.

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Thus $R_1^1 = R_2^1 = [0, \theta/2]$.

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Thus $R^2_1 = R^2_2 = [\theta/4, \theta/2]$.

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In general, after $2k$ iterations we have $R_i^{2k} = R_i^{2k} = [\ell_k, r_k]$ where

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- $\ell_k = \theta/3 - \left(\frac{1}{4}\right)^k \theta/3$
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Hence, $\lim_{k \to \infty} \ell_k = \lim_{k \to \infty} r_k = \theta/3$ and thus $(\theta/3, \theta/3)$ is the only rationalizable equilibrium.

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$$u_1(\theta/3, \theta/3) = u_2(\theta/3, \theta/3) = \theta^2/9$$

but

$$u_1(\theta/4, \theta/4) = u_2(\theta/4, \theta/4) = \theta^2/8$$

IESDS vs Rationalizability in Pure Strategies

Theorem 14
Assume that S is finite. Then for all k we have that $R_i^k \subseteq D_i^k$. That is, in particular, all rationalizable strategies survive IESDS.

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Then $s'_i \in G^{k+1}_{Rat}$ since s'_i is *not* eliminated from G^k_{Rat}.
However, since s_i is a best response to s_{-i} in G^{k+1}_{Rat}, we get $u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i})$.
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By induction hypothesis, s_i is a best response to s_{-i} in G and the claim has been proved.

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Keep in mind: If s_i is a best response to s_{-i} in G^k_{Rat}, then s_i is a best response to s_{-i} in G.

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By the claim, s_i is a best response to s_{-i} in G as well!
By induction hypothesis, $s_i \in R_i^{k+1} \subseteq R_i^k \subseteq D_i^k$ and $s_{-i} \in R_{-i}^k \subseteq D_{-i}^k$.

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(This follows from the fact that the “best response” relationship of s_i and s_{-i} is preserved by removing arbitrarily many other strategies.)
Thus s_i is not strictly dominated in G^k_{DS} and $s_i \in D^{k+1}_i$. □

Pinning Down Beliefs – Nash Equilibria

Criticism of previous approaches:

- Strictly dominant strategy equilibria often do not exist
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But are all strategy profiles really equally reasonable?

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(O, O) can be obtained as a profile where each player plays the best response to his belief and the **beliefs are correct**.

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Nash equilibrium can be defined as a set of beliefs (one for each player) and a strategy profile in which every player plays a best response to his belief and each strategy of each player is consistent with beliefs of his opponents.

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A usual definition is following:

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A pure-strategy profile $s^* = (s_1^*, \ldots, s_n^*) \in S$ is a (pure) Nash equilibrium if s_i^* is a best response to s_{-i}^* for each $i \in N$, that is

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Note that this definition is equivalent to the previous one in the sense that s_{-i}^* may be considered as the (consistent) belief of player i to which he plays a best response s_i^*

Nash Equilibria Examples

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In Cournot Duopoly, $(\theta/3, \theta/3)$ is the only Nash equilibrium.

(Best response relations: $q_1 = (\theta - q_2)/2$ and $q_2 = (\theta - q_1)/2$ are both satisfied only by $q_1 = q_2 = \theta/3$)

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Story:

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Two NE: (S, S), and (H, H), where the former is strictly better for each player than the latter! Which one is more reasonable?

Example: Stag Hunt

Strategy-form game model: $N = \{1, 2\}$, $S_1 = S_2 = \{S, H\}$, the payoff:

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This is supposed to explain that in real world there are societies that have similar endowments, access to technology and physical environment but have very different achievements, all because of self-fulfilling beliefs (or *norms* of behavior).

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Minimum secured by playing S is 0 as opposed to 3 by playing H
(We will get to this *minimax* principle later)

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So it seems to be rational to expect (H, H) (?)

Nash Equilibria vs Previous Concepts

Theorem 16

1. *If s^* is a strictly dominant strategy equilibrium, then it is the unique Nash equilibrium.*
2. *Each Nash equilibrium is rationalizable and survives IESDS.*
3. *If S is finite, neither rationalizability, nor IESDS creates new Nash equilibria.*

Proof: Homework!

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Proof: Homework!

Corollary 17

Assume that S is finite. If rationalizability or IESDS result in a unique strategy profile, then this profile is a Nash equilibrium.

Interpretations of Nash Equilibria

Except the two definitions, usual interpretations are following:

- When the goal is to give advice to all of the players in a game (i.e., to advise each player what strategy to choose), any advice that was not an equilibrium would have the unsettling property that there would always be some player for whom the advice was bad, in the sense that, if all other players followed the parts of the advice directed to them, it would be better for some player to do differently than he was advised. If the advice is an equilibrium, however, this will not be the case, because the advice to each player is the best response to the advice given to the other players.

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- When the goal is prediction rather than prescription, a Nash equilibrium can also be interpreted as a potential stable point of a dynamic adjustment process in which individuals adjust their behavior to that of the other players in the game, searching for strategy choices that will give them better results.